Simple circuit model of small tuned loop antenna including observable environmental effects

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Frequency swept input impedances and Q factors of different sized balanced tuned loop antennas have been measured over their tuning ranges. A simple circuit model, with a dominant radiation resistance proportional to frequency and loop area, agrees well with measurements. Conductor losses, dipole modes, and environmental effects are separately identifiable.

Introduction: The small tuned (magnetic) loop antenna typically consists of a single turn loop, tuned by a single capacitor, with a subsidiary input loop or gamma match. Over the HF 1.7–30 MHz frequency range loops with diameters of 0.8–1.2 m can have radiation efficiency of no worse than 90%, and can operate with powers of a few hundred watts. A loop with a copper tube diameter >20 to 30 mm can be capacitatively tuned over a ten-to-one frequency range at >90% efficiency. The tube diameter requirement means that efficient (transmitting) loops do not scale with frequency and are not useful above ~30–50 MHz. Typically a balanced loop has a small operational bandwidth, roughly proportional to loop size, corresponding to a Q factor of ~200–600.

The classic formula $3.12 \times 10^4 [A^2/\lambda^2]^2$ for the series radiation resistance of a small single turn tuned loop antenna is based on theory that is well founded and provably correct [1]. However, measurements show that it predicts radiation resistances which for loops $<\lambda/160$ in diameter are about a thousand times less than measured values. This shows that other radiation modes are more dominant in practice.

It has been proposed [2] that the small tuned loop also has a folded dipole radiation resistance, varying as the square of frequency, and this is larger than the classic loop mode at frequencies below the loop self resonant frequency. It has been shown by simulation [3] that a loop radiating an electric dipole or monopole mode together with a loop mode would be uni-directional to some extent. Practical measurements and published loop manufacturers claims [4] have since confirmed this.

Practical impedance measurements show that there are a number of other radiation modes and loss mechanisms with different frequency laws. Of these we find that the dominant radiation term for practical loops has a series radiation resistance proportional to frequency and loop area, with a magnitude that always exceeds the 'classic' loop radiation resistance below the loop self-resonant frequency.

The proposed model is shown in Fig. 1. It consists of a series tuned circuit with all possible radiation and loss resistances combined into a single *total* series resistance R_{tot}. A novel method of combination is proposed as follows. A gamma match is tapped on to the loop at a suitable point to give a near enough perfect match to 50 Ω. Whether twisted around the loop conductor or not, the gamma match can be represented by the combination of voltage feed to a tapping point and inductive (transformer) coupling to the tuned loop. A small coupling loop can also be modelled as inductive coupling with no tapping point. Both of these can be shown to be equivalent to a perfect transformer and a series inductance as indicated in Fig. 1.

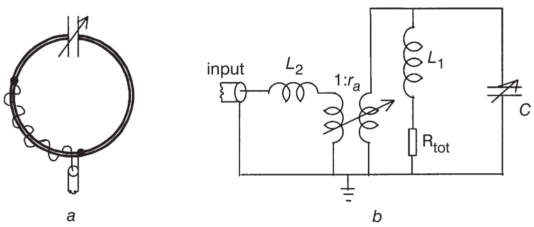

Fig. 1 *Typical loop configuration and proposed simple circuit model*
a Loop configuration *b* Circuit model

Fig. 2*a* shows the input impedance of this equivalent circuit plotted as a reflection coefficient against frequency on a Smith Chart for the set of component values found for a 70 cm diameter tuned loop made from 15 mm copper tubing. The loop was tuned to a resonant frequency of 10 MHz. Mathcad was used because Spice does not include a frequency dependent resistor as a standard component. The actual HP8547 network analyser results are shown in Fig. 2*b*. The results also closely match the model over the typical 5 to 25 MHz range of this antenna after the necessary component contributions to R_{tot} have been established by the following Q measurement method.

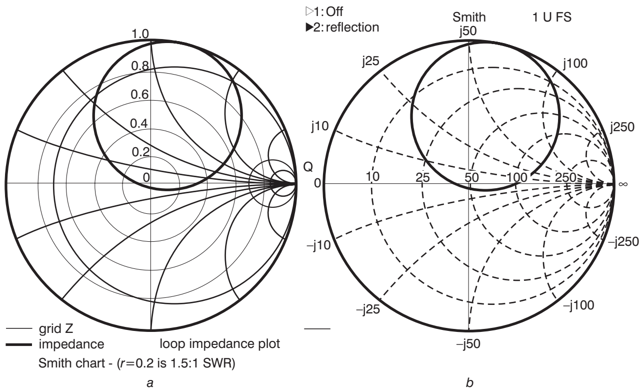

Fig. 2 *Comparison of measured and predicted input impedance at centre of loop tuning range (10.21 MHz)*
a Predicted *b* Measured

Measurement of Q: Accurate and consistent Q measurements over a wide loop tuning range are required so that the several contributions to R_{tot} may be separated and individually measured. Accurate Q values can be obtained from SWR measurements only if very short cable lengths are used between the source, SWR bridge and antenna. The HP 8457 network analyser can usefully compute out short cable lengths, but only if the cable dielectric and conductor losses are very small. The MFJ 259 portable antenna analyser has the advantage of small size.

A 'perfect' match is required for the measurement of the overall loop Q factor, Q_{BW}, by SWR. The twisted wire gamma match can nearly always provide this. It conveniently consists of a length of wire, being a small or large fraction of a half circumference, with a suitable tapping point found for this on the main loop using a crocodile clip or similar movable clamp. Wire spacing conveniently provides a means of fine adjustment.

Then the 3 dB bandwidth of a tuned circuit corresponds to a reflection coefficient of modulus $\rho = |(1-(1 \pm j))/(1+1 \pm j)| = 1/\sqrt{5}$ and this corresponds to an SWR of 2.62. Thus, Q_{meas} is obtained from measurements of the frequencies, f_1 and f_2, at which the SWR degrades to 2.62:1. Then $Q_{meas} = (f_1 + f_2)/2(f_1 - f_2)$. Variations in the impedance of practical coaxial cables mean that the measurements must be made as close to the antenna as possible.

Circuit model and comparison with results: The proposed circuit model and the parameters required to match it to practical Q measurements are now addressed. Fig. 3 compares the model and Q results for a 70 cm diameter loop made from 15 mm copper tubing measured at a lowest point height of 1.2 m over a wet field of Surrey clay. The required parameters are given as follows (SI units are used).

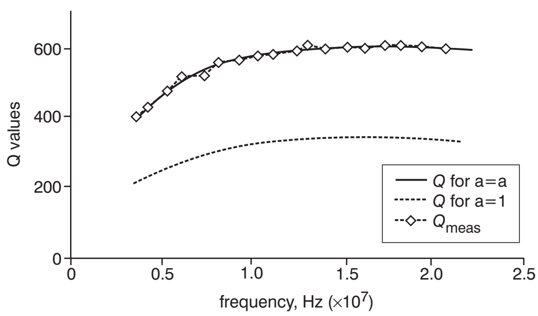

Fig. 3 *Comparison of simulated and measured Q values*

The simple circuit model of Fig. 1 has an input impedance, for $s = j\omega = 2\pi f$, given by

$$Z(s) = sL_2 + r_a^{-2}[(sL_1 + R_{tot})^{-1} + SC_1]^{-1} + sL_2 \quad (1)$$

L_1 is the loop inductance and the tuning capacitor C_1 sets the frequency of operation. r_a is the step-up ratio of the equivalent input transformer. The input inductance is L_2.

R_{tot} combines all radiation and loss resistances. In general, each of these varies with frequency differently. We propose the following combination with the selectable parameter a made equal to two.

$$R_{tot} = [R_{loop}^a + R_{trad}^a + R_{loss}^a + R_{env}^a + R_{ground}^a]^{1/a} \quad (2)$$

Where we have:

$$L_1 = 10^{-6} \times \pi D_{loop}^{1.25}(167\ D_{tube})^{-1/6} \quad (3)$$

$$Q_{ml} = Q_{il}\ D_{loop}^{-1} \qquad \text{with } Q_{il} = 300 \text{ to } 600 \quad (4)$$

$$R_{loop} = \omega L_1 / Q_{ml} \quad (5)$$

$$R_{trad} = \left(\frac{k_t\, f\ D_{loop}}{3 \times 10^8}\right)^4 \times 10\pi^6 \qquad \text{with } k_t = 1 \text{ traditionally} \quad (6)$$

$$R_{dip} = 200(k_{dip}\, f\ D_{loop}(\pi/2))^2 \qquad \text{with } k_{dip} = 1 \text{ to } 2 \text{ typically} \quad (7)$$

$$R_{loss} = 7.07 \times 10^{-6}\pi D_{loop}\, D_{tube}^{-1}(f_{MHz})^{0.5} \qquad \text{(for copper)} \quad (8)$$

$$R_{env} = k_e\ D_{loop}^2(f_{MHz})^{0.5} \quad k_e = 0.005 \text{ to } 0.65 \quad (9)$$

$$R_{ground} = k_c\ D_{loop}(1 + f^2 f_g^{-2}) \\ k_c = 0.02 \text{ to } 0.2 \text{ and } f_g = 2 \text{ to } 10\,\text{MHz} \quad (10)$$

$$Q_{BW} = \omega L_1 / R_{tot} \quad (11)$$

$$L_2 = \alpha L_1 = 0.1\, L_1 \text{ to } 1.1\, L_1 \quad (12)$$

Eqn. 3 has been found empirically to represent single turn loop inductances adequately over the range of loop dimensions so far considered.

Eqns. 4 and 5 for Q_{ml} is a novel proposal. Without this, it is not possible for Q_{BW} from the proposed model to be able to match the observed results for Q_{meas}. Note that the Q of the basic loop, Q_{ml}, does not vary with frequency, but it is inversely proportional to loop diameter, D_{loop}. The intrinsic loop Q factor Q_{il} can be considerably increased by an adjacent low-loss reflector (or in a screened room).

Eqn. 6 is the traditional book formula if $k_t = 1$. It is negligible unless the loop is tuned above its self resonant frequency. In practice it is sometimes useful to set this to a higher value, 2 to 3, because this can then represent the effect of a strong ground reflection on the loop impedance. (7) represents the balanced loop also acting as a folded dipole if $k_{dip} = 1$. If unbalanced and with a ground-plane, k_{dip} can rise to 2.

Eqn. 8 represents the copper tubing $f^{0.5}$ losses. Eqn. 9 represents $f^{0.5}$ losses in the environment from adjacent walls made for example from reinforced concrete or anechoic absorbent material.

Eqn. 10 represents a ground wave coupling effect that is found in practice. It appears as an approximately constant resistance down to a cut-off frequency f_g of typically 3 to 30 MHz. This term has been found to be essential to be able to match the model to the results for loops close to a real ground. It can become dominant for larger loops at low frequencies.

L_2 in (11) can be chosen empirically to give typically $\sim$20% improvement in frequency range of operation for a given SWR limit.

To obtain a fit for the results of Fig. 3 we chose $Q_{il} = 455$, $k_t = 1.1$, $k_{dip} = 1$, $k_e = 0.01$, $k_c = 0.016$, $f_g = 30$, $\alpha = 0.5$, $r_a = 23.2$.

An important finding shown in Fig. 4, is that model predictions cannot be matched to results with the parameter $a = 1$ in R_{tot} in (2). We find that $a = 2$, or a bit more, gives the best fit. The proposed explanation is that in general all the radiation and loss resistances are lightly coupled to each other and so RMS combination with $a = 2$ is appropriate. With $a = 2$, the agreement between the model and the measurements can be seen to be good.

Conclusions: A simple circuit model for balanced (and unbalanced) tuned loop antennas has been derived from SWR based Q measurements. The Q measurements are sufficiently accurate and sensitive to be able to identify and separate two (magnetic) loop radiation mechanisms, one electric (dipole) mode and the loop conductor losses. The Q measurement method has also been found to be sufficiently sensitive and accurate for the identification and measurement of ground and environmental effects. These effects can be losses, reflections or (re-)radiation from the ground or surrounding environment.

Acknowledgment: M. Harper has an EPSRC studentship with DSTL, Fort Halstead.

4 January 2002
Electronics Letters Online No: 20020673
DOI: 10.1049/el:20020673

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